



Subject: Grade 12 Mathematics

Topic: Term 2: Exam 1

Total: 129 Marks

1. B: $q = -2$

4 marks

Explanation:

$$g(x) = (x + 1)^2 - 2$$

The coordinates of the turning point are $(-1; -2)$.

The y -value of the turning point will be the asymptote of f .

$y = -2$ is the asymptote.

$$\therefore q = -2$$

2. 3 | $b=3$

3 marks

Explanation:

Substitute $(-3; \frac{1}{27})$ into f :

$$\frac{1}{27} = b^{-3} \quad \text{Solve for } b$$

$$\frac{1}{27} = \frac{1}{b^3}$$

$$\therefore 27 = b^3$$

$$b = 3$$

3. 4 | $a=4$

3 marks

Explanation:

Substitute $(16; 2)$ into $y = \log_a x$:

$$2 = \log_a 16 \quad \text{Convert to exponential form}$$

$$a^2 = 16 \quad \text{Solve for } a$$

$$\therefore a = 4$$



4. B: $0 < x \leq 1$

4 marks

Explanation:

To determine the equation of $f^{-1}(x)$, swap the x and y -values:

$$\begin{aligned}x &= \sqrt{y} \\x^2 &= y && \text{Square both sides} \\ \therefore f^{-1}(x) &= x^2\end{aligned}$$

Solve where $f^{-1}(x) = g(x)$ for $x > 0$:

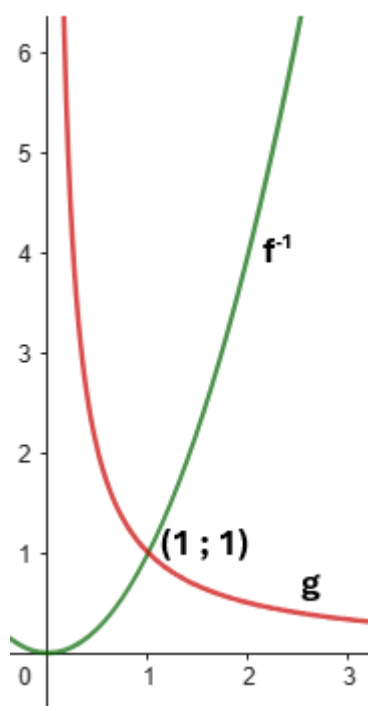
$$x^2 = \frac{1}{x}$$

$$x^3 = 1$$

$$x = 1$$

$f^{-1}(x)$ will be below $g(x)$ from the point of intersection to the asymptote of g .

Graphically it would look as follows:



$$\therefore f^{-1}(x) \leq g(x) \text{ for } 0 < x \leq 1.$$

Explanation:

- Each distance up is $0,9 \times$ previous down distance.

The following down distance will be the same as the up distance right before it.

Down: 2 ; 1,8 ; 1,62 ; ... constant $r = 0,9$

Up: 1,8 ; 1,62 ; 1,458 ; ... constant $r = 0,9$

(the first down of 2 m is the only difference in the sequences)

To determine the **fourth drop**, determine T_4 of the down sequence:

$$\begin{aligned} T_4 &= 2(0,9)^{4-1} \\ &= 1,458 \text{ m} \end{aligned}$$

- To determine the total distance travelled after the fourth down:

Add the 4 down distances to the 3 up distances OR multiply the 3 up distances by 2 (as there is an up and down that is the same) and then add the first down of 2 m.

$$\text{Sum of the down distances: (Use } S_n = \frac{a(r^n - 1)}{r - 1}\text{)}$$

$$\begin{aligned} S_3 &= \frac{1,8(0,9^3 - 1)}{0,9 - 1} \\ &= 4,878 \text{ m} \end{aligned}$$

There was a corresponding up for each of these down distances, so multiply by 2.

$$(4,878 \times 2) + 2 = 11,756 \text{ m}$$

- To determine the total distance travelled before the yo-yo comes to a stop, use a **sum to infinity** as there could be an infinite number of downs and ups. The ratio of 0,9 does satisfy the conditions of $-1 < r < 1$ for sum to infinity.

Do the **sum to infinity** for all the **up distances** and then **multiply by 2** for the corresponding down distances.

After this, **add the initial 2 m** down drop.

$$\begin{aligned} S_\infty &= \frac{1,8}{1 - 0,9} \\ &= 18 \end{aligned}$$

$$\text{Total distance} = 18 \times 2 + 2 = 38 \text{ m}$$

6. B: $x = 0,47$ or $x = 8,53$

4 marks

Explanation:

A quadratic equation must be in the form $ax^2 + bx + c = 0$ before solving for x .

$$-x^2 + 9x - 4 = 0$$

This cannot be factorised, so use the quadratic formula:

$$a = -1 ; b = 9 ; c = -4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Substitute}$$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(-1)(-4)}}{2(-1)} \quad \text{Simplify}$$

$$x = \frac{-9 \pm \sqrt{65}}{-2} \quad \text{Determine solutions}$$

$$x = 0,47 \text{ or } x = 8,53$$

7. 462

3 marks

Explanation:

The number of red squares as a number pattern is: 6 ; 12 ; 20 ; ...

If we extend the pattern by 2 terms it will be:

6 ; 12 ; 20 ; 30 ; 42 ; ...

This is going to be a quadratic pattern.

The **first differences** are 6 ; 8 ; 10 ; ...

This will give a constant **second difference** of 2.

To determine a , b and c :

$$2a = \text{second difference}$$

$$2a = 2$$

$$a = 1$$

$$3a + b = \text{first difference}$$

$$3(1) + b = 6$$

$$b = 3$$

$$a + b + c = \text{first term}$$

$$1 + 3 + c = 6$$

$$c = 2$$

Substitute the values of a , b and c :

$$T_n = n^2 + 3n + 2$$

$$\therefore T_{20} = (20)^2 + 3(20) + 2$$

$$T_{20} = 462$$

Explanation:

This is an **arithmetic** series because there is a **constant difference** of $\frac{3}{2}$ between consecutive terms.

Use $T_n = a + (n - 1)d$ to determine the position of the last term. This answer will give the **number of terms** in the series:

$$\frac{61}{2} = 2 + (n - 1)\left(\frac{3}{2}\right) \quad \text{Rearrange to solve for } n$$

$$\frac{57}{2} = (n - 1)\left(\frac{3}{2}\right)$$

$$19 = n - 1$$

$$n = 20$$

The last term is in position 20, therefore we need to calculate S_{20} :

$$S_n = \frac{n}{2}[2a + (n - 1)d] \quad \text{Substitute}$$

$$S_{20} = \frac{20}{2}[2(2) + (20 - 1)\left(\frac{3}{2}\right)]$$

$$S_{20} = 325$$

Explanation:

If it is sunshine today, then the tree diagram deals with the weather for tomorrow and the next day.

A is the probability of sunshine after a day of sunshine:

$$\therefore A = 0,8.$$

A and B must add up to 1 because they are complementary events:

$$\therefore B = 0,2.$$

C is the probability of sunshine after a day of sunshine:

$$\therefore C = 0,8.$$

C and D must add up to 1 because they are complementary events:

$$\therefore D = 0,2.$$

The probability of the 2 days sunshine after today is $0,8 \times 0,8 = 0,64$

10. C: $\frac{2}{9}$

4 marks

Explanation:

The number of students taking all three subjects is q .

Use the 71 students taking Accounting to determine q :

$$q + 13 + 20 + 8 = 71$$

$$q = 30$$

The total number of students is 135.

The probability when a student is chosen at random that the student take all three subjects:

$$P(\text{all three subjects}) = \frac{30}{135} = \frac{2}{9}$$

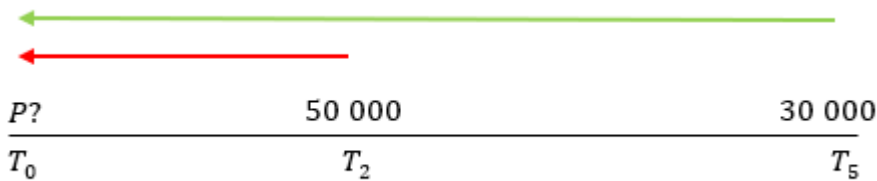
11. R62306,48 | R62306.48 | 62306,48 | 62306.48

3 marks

Explanation:

Use the timeline below to assist. Each sum of money represents the A value in

$$A = P(1 + i)^n.$$



To solve for P :

$$P = A(1 + i)^{-n} \text{ can be used.}$$

n represents months so multiply the years by 12.

$$P = 50\,000\left(1 + \frac{0,0825}{12}\right)^{-(2 \times 12)} + 30\,000\left(1 + \frac{0,0825}{12}\right)^{-(5 \times 12)}$$

$$= \text{R}62\,306,48$$

Explanation:

• The **sample space** is 250 people. The sample space is the value at the bottom right-hand corner.

• The **probability** that a person has **brown hair** is $\frac{110}{250} = \frac{11}{25}$.

• The **probability** that a person has **blue eyes** is $\frac{175}{250} = \frac{7}{10}$.

• To determine whether the events are independent:

$$\begin{aligned} & P(\text{Brown Hair}) \times P(\text{Blue Eyes}) \\ &= \frac{11}{25} \times \frac{7}{10} \\ &= \frac{77}{250} \end{aligned}$$

$$\begin{aligned} & P(\text{Brown Hair and Blue Eyes}) \\ &= \frac{80}{250} \\ &= \frac{8}{25} \end{aligned}$$

$$P(\text{Brown Hair and Blue Eyes}) \neq P(\text{Brown Hair}) \times P(\text{Blue Eyes})$$

Brown hair and blue eyes are **not independent** events.

Explanation:

$$\begin{aligned} 4^x + 16 &= 10 \cdot 2^x && \text{Change the 4 to a prime base} \\ 2^{2x} + 16 &= 10 \cdot 2^x && \text{Equate to zero} \\ 2^{2x} - 10 \cdot 2^x + 16 &= 0 \end{aligned}$$

The exponents are in a 2 : 1 ratio. This is the same as a quadratic trinomial. Let $2^x = k$ as this will be easier to factorise.

$$\begin{aligned} k^2 - 10k + 16 &= 0 && \text{Factorise} \\ (k - 2)(k - 8) &= 0 && \text{Solve for } k \\ k &= 2 \text{ or } k = 8 \end{aligned}$$

Now if $2^x = k$, solve for x :

$$\begin{aligned} 2^x &= 2 && \text{or } 2^x = 8 \\ 2^x &= 2^1 && 2^x = 2^3 \\ \therefore x &= 1 && \text{or } x = 3 \end{aligned}$$

14. $2 \mid \log_2 x \mid x > 0$

6 marks

Explanation:

$(2 ; 4)$ lies on f :

$$4 = b^2$$

$$2 = b$$

The value of b is 2.

$$\therefore f(x) = 2^x$$

The inverse is a reflection over the line $y = x$:

$$x = 2^y \quad \text{Swap } x \text{ and } y \text{ for the inverse}$$

$$\log_2 x = y \quad \text{Make } y \text{ the subject of the formula}$$

$$\therefore f^{-1}(x) = \log_2 x \quad \text{Inverse of } f$$

The domain of $f^{-1}(x)$ can be interpreted from f .

The range of f is $y > 0$.

With an inverse x and y swap, so the **domain of the inverse** then becomes the **range of the function**.

Therefore the domain of $f^{-1}(x)$ will be $x > 0$.

15. A: $h(x) = 2^x$

4 marks

Explanation:

The rule for reflection in the y -axis: $(x ; y) \rightarrow (-x ; y)$

To reflect over the y -axis, change the sign of the x -value:

$$y = \left(\frac{1}{2}\right)^{-x} \quad \text{Use laws of exponents to change to a positive exponent}$$

$$y = 2^x$$

$$\therefore h(x) = 2^x$$

16. C | C | B | A

8 marks

Explanation:

Graph A is the inverse of graph C .

A and C are reflections of one another over the line $y = x$ (the dotted line). This is an inverse.

Graph B is a reflection of graph C over the y -axis.

B and C are mirror images of one another over the y -axis.

Graph B is a decreasing function.

From left to right on the x -axis, the y -values of B decrease.

Graph A has a domain $x \in (0 ; \infty)$.

17. choose the lowest common denominator | $5 - 2\sqrt{3}$ | the conjugate | $(9 - \sqrt{3})/6$

8 marks

Explanation:

$$\frac{\sqrt{3}}{\sqrt{3} - 1} - \frac{2}{\sqrt{3}} \quad \text{Choose the lowest common denominator}$$

$$= \frac{\sqrt{3} \cdot \sqrt{3} - 2(\sqrt{3} - 1)}{\sqrt{3}(\sqrt{3} - 1)} \quad \text{Simplify}$$

$$= \frac{3 - 2\sqrt{3} + 2}{3 - \sqrt{3}}$$

$$= \frac{5 - 2\sqrt{3}}{3 - \sqrt{3}}$$

The **conjugate** is the denominator with the opposite sign:

$$3 + \sqrt{3}$$

Multiply the expression by 1 with using the conjugate: $\frac{3 + \sqrt{3}}{3 + \sqrt{3}}$

$$= \frac{5 - 2\sqrt{3}}{3 - \sqrt{3}} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}} \quad \text{Multiply the numerator and denominator}$$

$$= \frac{15 + 5\sqrt{3} - 6\sqrt{3} - 2\sqrt{3}\sqrt{3}}{9 - \sqrt{3}\sqrt{3}} \quad \text{Simplify}$$

$$= \frac{15 - \sqrt{3} - 6}{9 - 3}$$

$$= \frac{9 - \sqrt{3}}{6}$$

18. 19 | p=19

3 marks

Explanation:

$$\sum_{k=1}^p (2k - 1) < 400$$

If you add a certain number of terms in this series, the answer must be less than 400.

Substitute $k = 1 ; 2 ; 3 ; \dots$ into $(2k - 1)$ to determine the first 3 terms of the series:

$1 + 3 + 5$ are the **first three terms** of an **arithmetic series** with a **common difference** of 2.

Solve the S_n as an inequality, and then determine the whole number that will satisfy the condition for the sum to be less than 400.

Use the formula: $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$\frac{n}{2}[2(1) + (n - 1)(2)] < 400$$

$$n[2 + 2n - 2] < 800$$

$$2n^2 - 800 < 0$$

$$n^2 - 400 < 0$$

$$(n - 20)(n + 20) < 0$$

Use the table method or any other method you have learnt in Grade 11 to solve the quadratic inequality.

The **solution** is: $-20 < n < 20$.

n is a number of terms which cannot be negative.

The part of the solution that would apply is only the positive numbers: $0 < n < 20$.

The greatest number of terms that would give a solution less than 400 is therefore $n = 19$.

19 terms are added together. In sigma notation,

"top - bottom + 1 = number of terms"

$$p - 1 + 1 = 19$$

$$p = 19$$

19. $A(1; 3) \mid (1; 3) \mid 1; 3$

4 marks

Explanation:

$$g(x) = x^2 + bx + 8$$

The turning point of g lies on the asymptote of f , which is $y = -1$.

The axis of symmetry of g is $x = 3$.

Therefore the turning point is $(3; -1)$.

Substitute the turning point into g to solve for b :

$$-1 = 3^2 + 3b + 8$$

$$-3b = 18$$

$$b = -6$$

$$\therefore g(x) = x^2 - 6x + 8$$

• A is the point of intersection of f and g :

$$x^2 - 6x + 8 = 2.2^x - 1$$

It is not possible to solve this equation algebraically, but it possible by trial-and-error to determine an x -value that will satisfy both sides of the equation.

By substituting an x -value into f and g , you want to get the same answer for both.

This will then be the y -value.

The x -value will be smaller than 3 because it is before the axis of symmetry.

Try $x = 1$:

$$g(1) = (1)^2 - 6(1) + 8 = 3 \quad \text{and} \quad f(1) = 2.2^1 - 1 = 3$$

$\therefore x = 1$ satisfies the equation because it generates equal y -values of 3.

$$\therefore A(1; 3)$$

20. $x=7 \mid 7 \mid 7=x$

3 marks

Explanation:

$$\sqrt{x+2} + 4 = x$$

$$\sqrt{x+2} = x - 4$$

Isolate the surd

$$x + 2 = (x - 4)^2$$

Square both sides

$$x + 2 = x^2 - 8x + 16$$

Simplify

$$0 = x^2 - 9x + 14$$

Equate to zero

$$0 = (x - 7)(x - 2)$$

Factorise

$$x = 7 \text{ or } x = 2$$

Solve for x

Check each solution by substituting into the step where the surd is isolated:

For $x = 7$:

$$\text{LHS: } \sqrt{7+2} = \sqrt{9} = 3$$

$$\text{RHS: } 7 - 4 = 3$$

$$\therefore x = 7$$

For $x = 2$:

$$\text{LHS: } \sqrt{2+2} = \sqrt{4} = 2$$

$$\text{RHS: } 2 - 4 = -2$$

$$\therefore x \neq 2$$

The solution is $x = 7$

21. C: $k = -3$ or $k = 2$

4 marks

Explanation:

A geometric sequence has a constant ratio:

$$\frac{T_3}{T_2} = \frac{T_2}{T_1}$$

$$\frac{k-6}{-k} = \frac{-k}{-1} \quad \text{Substitute}$$

$$-(k-6) = k^2 \quad \text{Cross-multiply}$$

$$-k + 6 = k^2$$

$$0 = k^2 + k - 6 \quad \text{Equate to 0}$$

$$0 = (k+3)(k-2) \quad \text{Factorise}$$

$$k = -3 \text{ or } k = 2$$

Substitute both value(s) for k to determine whether the sequence is geometric or not:

If $k = -3$:

$-1 ; 3 ; -9 ; \dots$

This is a geometric sequence with a constant ratio of -3.

If $k = 2$:

$-1 ; -2 ; -4 ; \dots$

This is a geometric sequence with a constant ratio of 2.

Therefore both $k = -3$ or $k = 2$ generate a geometric sequence.

Explanation:

The **horizontal asymptote** of g goes through A , the **y -intercept** of f .

Determine A by calculating $f(0)$:

$$\begin{aligned} f(0) &= 2(0) + 3 \\ &= 3 \\ A(0; 3) \end{aligned}$$

The horizontal asymptote of g is $y = 3$, therefore the value of q is 3.

B is the **point of intersection** of f and g .

For a point of intersection, let $f(x) = g(x)$:

$$2x + 3 = \frac{4}{x - 1} + 3 \quad \text{Simplify}$$

$$2x = \frac{4}{x - 1} \quad \text{Multiply by } x - 1$$

$$\begin{aligned} 2x(x - 1) &= 4 \\ 2x^2 - 2x - 4 &= 0 \\ x^2 - x - 2 &= 0 \\ (x - 2)(x + 1) &= 0 \\ x &= 2 \text{ or } x = -1 \end{aligned}$$

B is on the **negative** side of the x -axis, therefore $x = -1$.

Determine $f(-1)$:

$$\begin{aligned} f(-1) &= 2(-1) + 3 \\ &= 1 \\ \therefore B(-1; 1) \end{aligned}$$

$$23. C: f^{-1}(x) = \frac{x + 3}{2}$$

4 marks

Explanation:

$f^{-1}(x)$ will be the **inverse** of $f(x)$. An inverse is a reflection over the line $y = x$, and we swap the x and y .

$$\begin{aligned} x &= 2y - 3 \\ x + 3 &= 2y && \text{Rearrange to make } y \text{ the subject of the formula} \\ \frac{x + 3}{2} &= y \end{aligned}$$

$$\therefore f^{-1}(x) = \frac{x + 3}{2}$$

Explanation:

It is given that it is an arithmetic sequence. Substitute into $T_n = a + (n - 1)d$:

$$T_3 = 7$$

$$7 = a + (3 - 1)d$$

$$7 = a + 2d \quad \dots (1)$$

$$T_{12} = 25$$

$$25 = a + (12 - 1)d$$

$$25 = a + 11d \quad \dots (2)$$

$$a = 25 - 11d \quad \dots (3)$$

Substitute (3) into (1):

$$7 = 25 - 11d + 2d$$

$$-18 = -9d$$

$$d = 2$$

Substitute $d = 2$ into (3):

$$a = 25 - 11(2)$$

$$a = 3$$

There is enough information now to substitute into $S_n = \frac{n}{2}[2a + (n - 1)d]$:

$$\begin{aligned} S_{30} &= \frac{30}{2}[2(3) + (30 - 1)(2)] \\ &= 960 \end{aligned}$$

Explanation:

$$\sum_{k=2}^4 (2^k - 1)$$

Substitute $k = 2 ; 3 ; 4$

$$k = 2 : 2^2 - 1 = 3$$

$$k = 3 : 2^3 - 1 = 7$$

$$k = 4 : 2^4 - 1 = 15$$

The **first three terms** of the series are $3 + 7 + 15$.

To determine the number of terms in sigma notation:

"top - bottom + 1"

$$4 - 2 + 1 = 3$$

There are 3 terms in the series.

The **value of the series** is $3 + 7 + 15 = 25$.

26. 11,8% | 12,4%

4 marks

Explanation:

Nominal rate to effective rate:

$$i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n - 1$$

$$i_{eff} = \left(1 + \frac{0,112}{12}\right)^{12} - 1 \quad \text{Substitute}$$

$$i_{eff} = 0,1179... \quad \text{Multiply by 100}$$

$$= 11,8\% \text{ per annum}$$

Effective rate to nominal rate:

$$0,13 = \left(1 + \frac{i_{nom}}{4}\right)^4 - 1 \quad \text{Substitute}$$

$$1,13 = \left(1 + \frac{i_{nom}}{4}\right)^4 \quad \text{Rearrange}$$

$$\sqrt[4]{1,13} - 1 = \frac{i_{nom}}{4}$$

$$i_{nom} = (\sqrt[4]{1,13} - 1) \times 4$$

$$i_{nom} = 0,124... \quad \text{Multiply by 100}$$

$$= 12,4\% \text{ per annum compounded quarterly}$$

27. 0,2 | 2/10 | 1/5

4 marks

Explanation:

The probability identity is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substitute:

$$0,9 = 0,45 + 0,65 - P(A \cap B)$$

$$0,9 = 1,1 - P(A \cap B)$$

$$P(A \cap B) = 1,1 - 0,9$$

$$P(A \cap B) = 0,2$$

28. $x \leq -5$ or $x \geq 4$; $x \neq 6$ | $x = 6$

4 marks

Explanation:

$$\frac{\sqrt{x^2 + 4x - 5}}{x - 6}$$

a. For the expression to be **real**, firstly the **denominator cannot be zero**:

$$x - 6 \neq 0$$

$$x \neq 6$$

And the value **under the root** needs to be **positive**:

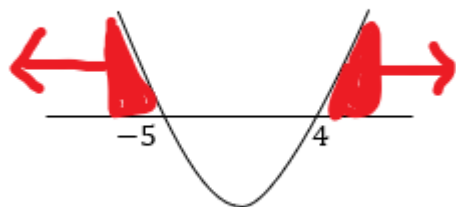
$$x^2 + 4x - 5 \geq 0 \quad \text{Solve the inequality}$$

$$(x + 5)(x - 4) \geq 0 \quad \text{Factorise}$$

The critical values are -5 and 4.

Either use the table method or quadratic method to interpret solutions.

Below is the quadratic method:



$$x \leq -5 \text{ or } x \geq 4$$

$$\therefore x \leq -5 \text{ or } x \geq 4 ; x \neq 6$$

b. The expression will be undefined when the denominator is equal to zero.

$$x - 6 = 0$$

$$\therefore x = 6$$

Total: 129 Marks